

FIG. 1

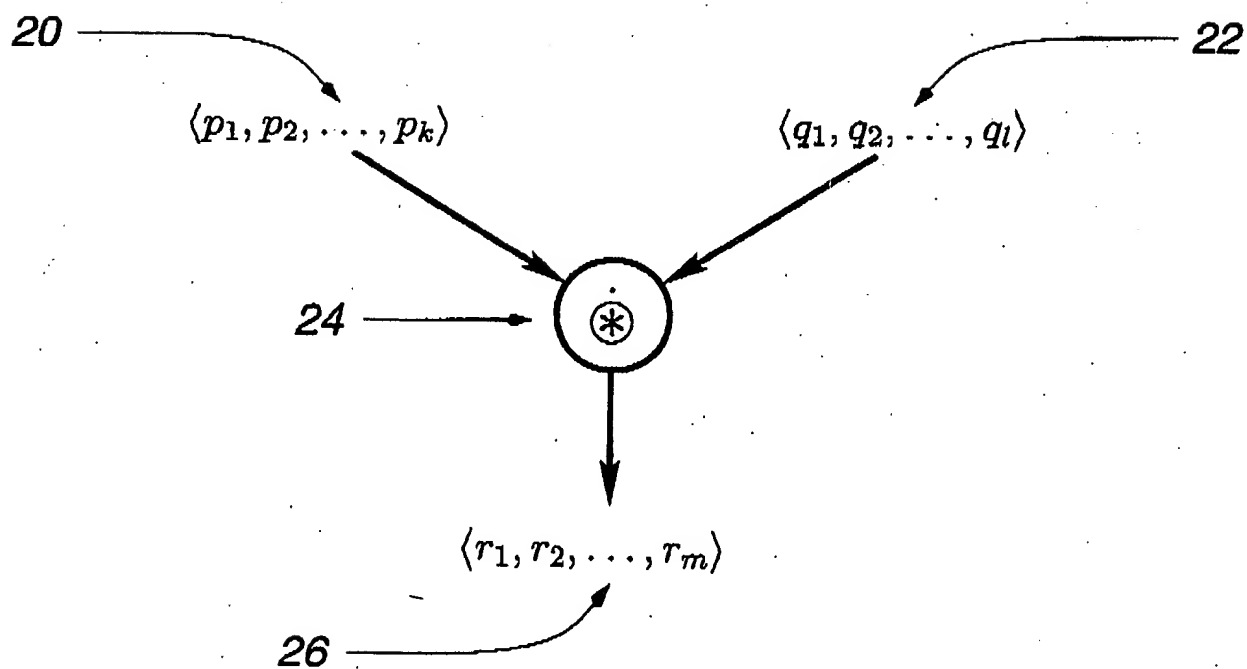


FIG. 2

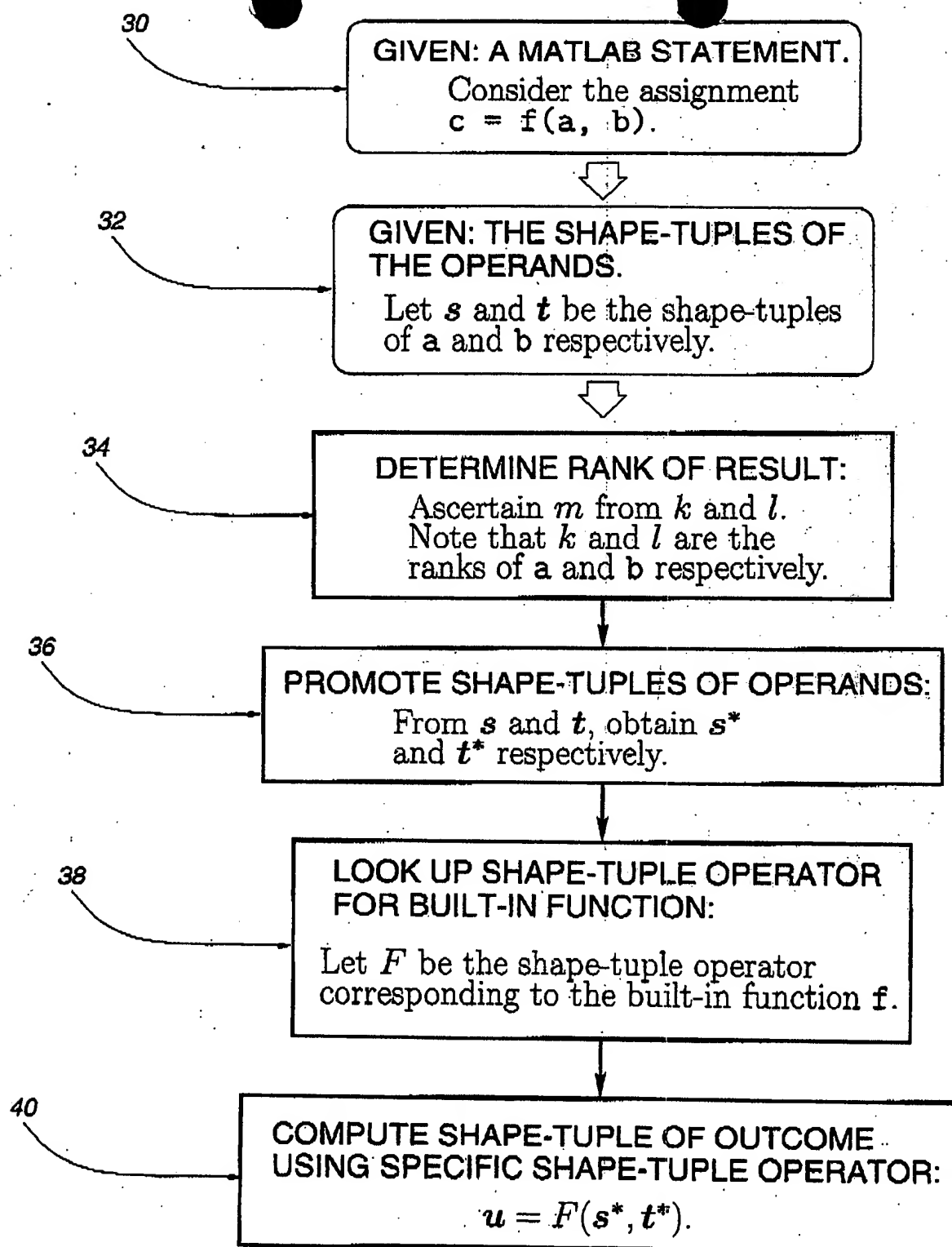


FIG. 3

Operator	Rank
$a*b$ $a+b$ $a-b$ $a.*b$ $a.^b$ $a./b$ $a.\backslash b$ $a==b$ $a\sim b$ $a<b$ $a>b$ $a<=b$ $a>=b$ $a\&b$ $a b$ a/b $a\backslash b$ $[a, b]$ $[a; b]$	$\max(R(a), R(b))$
$+a$ $-a$ $\sim a$ a' $a.'$	$R(a)$
$c(:) \leftarrow a$	$R(c)$
a^b $a(:)$ rand	2
$a:b$ $\text{rand}(a, b)$ $\text{ones}(a, b)$	2
$a(e)$	$R(e)$
$a(e_1, e_2, \dots, e_n)$ $\text{rand}(e_1, e_2, \dots, e_n)$ $\text{ones}(e_1, e_2, \dots, e_n)$	n
$c(e) \leftarrow a$	$\max(R(a), R(c))$
$c(e_1, e_2, \dots, e_n) \leftarrow a$	$\max(n, R(c))$

FIG. 4

MATLAB Expression c	Shape Expression	$\bar{\theta}(e)$	u
a+b	$s \oplus t$	$\bar{\theta}(a)\bar{\theta}(b)(1 - (1 - \bar{\alpha}(a))$ $(1 - \bar{\alpha}(b))(1 - \bar{\beta}(a)\bar{\beta}(b))$ $\delta(\Psi s \Psi \Gamma_1 - t \Gamma_1))$	$(1 - \bar{\theta}(e))\pi^* + \bar{\theta}(e)(s^* \bar{\alpha}(b) +$ $t^* \bar{\alpha}(a)(1 - \bar{\alpha}(b)) + (s^* \Gamma_1 +$ $t^* \Gamma_2 + I - \Gamma_1 - \Gamma_2)(1 - \bar{\alpha}(a))$ $(1 - \bar{\alpha}(b)))$
a+b a-b a.*b a.^b a./b a.\b a==b a~=b a<b a>b a<=b a>=b a&b a\b	$s \oplus t$	$\bar{\theta}(a)\bar{\theta}(b)(1 - (1 - \bar{\alpha}(a))$ $(1 - \bar{\alpha}(b))(1 - \delta(s - t)))$	$(1 - \bar{\theta}(e))\pi^* + \bar{\theta}(e)(s^* \bar{\alpha}(b) +$ $t^*(1 - \bar{\alpha}(b)))$
+a -a ~a	$\wr s$	$\bar{\theta}(a)$	s^*
a~b	$s \odot t$	$\bar{\theta}(a)\bar{\theta}(b)(1 - (1 - \bar{\alpha}(a)\bar{\beta}(b))$ $\delta(t \Gamma_1 - \Psi t \Psi \Gamma_1))$ $(1 - \bar{\alpha}(b)\bar{\beta}(a)\delta(s \Gamma_1 -$ $\Psi s \Psi \Gamma_1)))$	$(1 - \bar{\theta}(e))\pi^* + \bar{\theta}(e)$ $(s^* \bar{\alpha}(b) + t^*(1 - \bar{\alpha}(b)))$
a' a.'	$\neg s$	$\bar{\beta}(a)$	$(1 - \bar{\theta}(e))\pi^* + \bar{\theta}(e)\Psi s^* \Psi$
a/b	$s \oslash t$	$\bar{\theta}(a)\bar{\theta}(b)(1 - \bar{\alpha}(b))(1 - \bar{\alpha}(a))$ $(1 - \bar{\beta}(b))(1 - \bar{\beta}(a)\bar{\beta}(b))$ $\delta(s \Gamma_2 - t \Gamma_2)$	$(1 - \bar{\theta}(e))\pi^* + \bar{\theta}(e)(s^* \bar{\alpha}(b) +$ $t^*(1 - \bar{\beta}(b)) + (s^* \Gamma_1 + I -$ $\Gamma_1 - \Gamma_2 + \Psi t^* \Psi \Gamma_2)(1 - \bar{\alpha}(b))\bar{\beta}(b))$
a\b	$s \dot{\oslash} t$	$\bar{\theta}(a)\bar{\theta}(b)(1 - \bar{\alpha}(a))(1 - \bar{\alpha}(b))$ $(1 - \bar{\beta}(a))(1 - \bar{\beta}(a)\bar{\beta}(b))$ $\delta(s \Gamma_1 - t \Gamma_1)$	$(1 - \bar{\theta}(e))\pi^* + \bar{\theta}(e)(t^* \bar{\alpha}(a) +$ $s^*(1 - \bar{\beta}(a)) + (\Psi s^* \Psi \Gamma_1 + I -$ $\Gamma_1 - \Gamma_2 + t^* \Gamma_2)(1 - \bar{\alpha}(a))\bar{\beta}(a))$
[a; b]	$s \oplus t$	$\bar{\theta}(a)\bar{\theta}(b)\delta(s(I - \Gamma_1) -$ $t(I - \Gamma_1))$	$(1 - \bar{\theta}(e))\pi^* + \bar{\theta}(e)$ $(s^* + t^* \Gamma_1)$
[a, b]	$s \oplus t$	$\bar{\theta}(a)\bar{\theta}(b)\delta(s(I - \Gamma_2) -$ $t(I - \Gamma_2))$	$(1 - \bar{\theta}(e))\pi^* + \bar{\theta}(e)$ $(s^* \Gamma_2 + t^*)$

FIG. 5

Shape-Tuple Class Operator	Identity	Associativity	Commutativity	Idempotent Law
\oplus	i	\times	\times	\times
\oplus	i	\checkmark	\checkmark	\checkmark
\wr	$-$	$-$	$-$	$-$
\odot	i	\checkmark	\checkmark	\times
\lrcorner	$-$	$-$	$-$	$-$
\oslash	\times	\times	\times	\times
\circ	\times	\times	\times	\times
\odot	\times	\checkmark	\checkmark	\times
\ominus	\times	\checkmark	\checkmark	\times

FIG. 6